

Notes de correction du devoir de contrôle du premier semestre

Mécanique des Solides Indéformables

Exercice 1 :

1. Le plan $(O, \vec{y}, \vec{z})$ est un plan de symétrie matérielle pour la porte $\rightarrow G \in (O, \vec{y}, \vec{z}) \rightarrow$

$$\vec{OG} = y_G \vec{y} + z_G \vec{z}$$

- pour $z = -\frac{L}{2}$, le plan $(\vec{x}, \vec{y})$ est un plan de symétrie matérielle $\rightarrow z_G = -\frac{L}{2}$

- $y_G = \frac{m_1 y_{G_1} + m_2 y_{G_2}}{m_1 + m_2}$ avec :

• $m_1 = \sigma L l$; $y_{G_1} = -R \cos \alpha$

• $m_2 = \rho V_2$ où $V_2 = \int_{-L}^0 \int_{-\alpha}^{\alpha} \int_{R-e}^R r dr d\theta dz = \alpha L (R^2 - (R-e)^2)$

• $y_{G_2} = \frac{1}{V_2} \int_{-L}^0 \int_{-\alpha}^{\alpha} \int_{R-e}^R -r \cos \theta r dr d\theta dz = -\frac{2 \sin \alpha}{3 \alpha} \left(\frac{R^3 - (R-e)^3}{R^2 - (R-e)^2} \right)$

d'où $y_G = -\frac{3\sigma l R \cos \alpha + 2\rho(R^3 - (R-e)^3) \sin \alpha}{\sigma l + \rho \alpha (R^2 - (R-e)^2)}$

2. Le plan $(O, \vec{y}, \vec{z})$ est un plan de symétrie matérielle pour la porte $\rightarrow (O, \vec{x})$ est un axe

principal d'inertie $\rightarrow E_O = F_O = 0 \rightarrow [I_O(Porte)] = \begin{bmatrix} A_O & 0 & 0 \\ 0 & B_O & -D_O \\ 0 & -D_O & C_O \end{bmatrix}_{(\vec{x}, \vec{y}, \vec{z})}$

3. $I_{G_1 \vec{z}}(S_1) = \int_{P \in S_1} (x^2 + y^2) dm$ avec $y = 0 \rightarrow I_{G_1 \vec{z}}(S_1) = \frac{m_1}{lL} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{l}{2}}^{\frac{l}{2}} x^2 dx dz = \frac{m_1 l^2}{12}$

4. $I_{O \vec{z}}(S_2) = \int_{P \in S_2} (x^2 + y^2) dm = \frac{m_2}{V_2} \int_{-L}^0 \int_{-\alpha}^{\alpha} \int_{R-e}^R r^2 r dr d\theta dz = \frac{m_2}{2} \left(\frac{R^4 - (R-e)^4}{R^2 - (R-e)^2} \right)$

5. $I_{O \vec{z}}(Porte) = I_{O \vec{z}}(S_1) + I_{O \vec{z}}(S_2) = I_{G_1 \vec{z}}(S_1) + m_1 R^2 \cos^2 \alpha + I_{O \vec{z}}(S_2) \rightarrow$

$$I_{O \vec{z}}(Porte) = m_1 \left(\frac{l^2}{12} + R^2 \cos^2 \alpha \right) + \frac{m_2}{2} \left(\frac{R^4 - (R-e)^4}{R^2 - (R-e)^2} \right)$$

Exercice 2 :

1. $\vec{V}(A \in 2/1) = \frac{d\vec{OA}}{dt} \Big|_1 = \dot{\lambda} \vec{x}_1$

2. $\vec{V}(A \in 2/0) = \vec{V}(A \in 2/1) + \vec{V}(A \in 1/0) = \vec{V}(A \in 2/1) + \underbrace{\vec{V}(O \in 1/0)}_{=\vec{0}} + \underbrace{\vec{\Omega}_{(1/0)}}_{=\dot{\theta} \vec{z}_0} \wedge \vec{OA}$

$\rightarrow \vec{V}(A \in 2/0) = \dot{\lambda} \vec{x}_1 + \dot{\theta} \vec{y}_1$

3. $\vec{V}(C \in 3/0) = \vec{V}(C \in 3/0) + \vec{\Omega}_{(3/0)} \wedge \vec{AC} = \underbrace{\vec{V}(A \in 3/2)}_{=\vec{0}} + \vec{V}(A \in 2/0) + \underbrace{\vec{\Omega}_{(3/0)}}_{=\dot{\beta} \vec{z}_0} \wedge \vec{AC}$

$\rightarrow \vec{V}(C \in 3/0) = \dot{\lambda} \vec{x}_1 + \dot{\theta} \vec{y}_1 - L \dot{\beta} \vec{x}_3$

$$4. \vec{\Omega}_{(5/0)} = \vec{0} \text{ (glissière)} ; \vec{V}(F \in 5/0) = \left. \frac{d\vec{DF}}{dt} \right|_0 = -\dot{\eta} \vec{y}_0$$

$$\boxed{\{\mathcal{V}_{(5/0)}\}_F = \begin{Bmatrix} \vec{0} \\ -\dot{\eta} \vec{y}_0 \end{Bmatrix}}$$

$$5. \vec{V}(C \in 3/5) = \vec{0} \rightarrow \vec{V}(C \in 3/0) = \vec{V}(C \in 5/0) = \vec{V}(F \in 5/0) \text{ car } \vec{\Omega}_{(5/0)} = \vec{0} \\ \rightarrow \dot{\lambda} \vec{x}_1 + \dot{\lambda} \dot{\theta} \vec{y}_1 - L \dot{\beta} \vec{x}_3 = -\dot{\eta} \vec{y}_0$$

$$\begin{cases} \vec{x}_1 = \cos \theta \vec{x}_0 + \sin \theta \vec{y}_0 \\ \vec{y}_1 = -\sin \theta \vec{x}_0 + \cos \theta \vec{y}_0 \\ \vec{x}_3 = \cos \beta \vec{x}_0 + \sin \beta \vec{y}_0 \end{cases} \Rightarrow \begin{cases} \dot{\lambda} \cos \theta - \dot{\lambda} \dot{\theta} \sin \theta - L \dot{\beta} \cos \beta = 0 & (1) \\ \dot{\lambda} \sin \theta + \dot{\lambda} \dot{\theta} \cos \theta - L \dot{\beta} \sin \beta = -\dot{\eta} & (2) \end{cases}$$

$$6. \vec{V}(B \in 3/0) = \vec{V}(C \in 3/0) + \vec{\Omega}_{(3/0)} \wedge \vec{CB} = -\dot{\eta} \vec{y}_0 + a \dot{\beta} \vec{x}_3 \\ \rightarrow \boxed{\vec{V}(B \in 3/0) = a \dot{\beta} \cos \beta \vec{x}_0 + (a \dot{\beta} \sin \beta - \dot{\eta}) \vec{y}_0}$$

$$7. \vec{\Omega}_{(4/0)} = \dot{\alpha} \vec{z}_0 ; \vec{V}(E \in 4/0) = \vec{0}$$

$$\boxed{\{\mathcal{V}_{(4/0)}\}_E = \begin{Bmatrix} \dot{\alpha} \vec{z}_0 \\ \vec{0} \end{Bmatrix}}$$

$$\vec{V}(B \in 4/0) = \vec{V}(E \in 4/0) + \vec{\Omega}_{(4/0)} \wedge \vec{EB} \rightarrow \boxed{\vec{V}(B \in 4/0) = -b \dot{\alpha} \vec{x}_4}$$

$$8. \vec{V}(B \in 3/4) = \vec{0} \rightarrow \vec{V}(B \in 3/0) = \vec{V}(B \in 4/0)$$

$$\vec{x}_4 = \cos \alpha \vec{x}_0 + \sin \alpha \vec{y}_0 \Rightarrow \begin{cases} a \dot{\beta} \cos \beta = -b \dot{\alpha} \cos \alpha & (3) \\ a \dot{\beta} \sin \beta - \dot{\eta} = -b \dot{\alpha} \sin \alpha & (4) \end{cases}$$

$$9. \begin{cases} -\dot{\lambda} \dot{\theta} + L \frac{\dot{\beta}}{2} = 0 \\ \dot{\lambda} - L \frac{\dot{\beta} \sqrt{3}}{2} = -\dot{\eta} \\ -a \frac{\dot{\beta}}{2} + b \dot{\alpha} \frac{\sqrt{3}}{2} = 0 \\ a \frac{\dot{\beta} \sqrt{3}}{2} - \dot{\eta} = -b \frac{\dot{\alpha}}{2} \end{cases} \Rightarrow \boxed{\dot{\lambda} = \frac{7}{8} \dot{\eta}}$$

$$10. \dot{\eta} \geq 20 \text{ cm/s et } \dot{\lambda} = \frac{7}{8} \dot{\eta} \Rightarrow \dot{\lambda} \geq 17,5 \text{ cm/s} \Rightarrow \boxed{\dot{\lambda}_{min} = 17,5 \text{ cm/s}}$$