

Correction devoir de contrôle

nr PC 2 : 2018 / 2019

Exercice 1:

1) $n_x > n_y \Rightarrow n_x > n_y$: l'onde incidente : onde lente
ou $v = \frac{c}{n}$ et n_y : onde rapide

2) $\vec{E}(z, t) = E_0 (\hat{u}_x + \alpha e^{j\varphi} \hat{u}_y) e^{j(\omega t - k_0 z)}$
 $\varphi = 0, \varphi = \pi$: rectiligne ; $\alpha = 1$ et $\varphi = \frac{\pi}{2}$ ou $\varphi = \frac{3\pi}{2}$
 $\varphi = \frac{\pi}{2}, \varphi = \frac{3\pi}{2}$: elliptique ; $\alpha = 1$ et $\varphi = \frac{\pi}{2}$ ou $\varphi = \frac{3\pi}{2}$: circulaire.

3) $\vec{E}_{trans} = \begin{cases} E_0 e^{j(\omega t - k_0(z-e) - n_x k_0 e)} \\ \alpha E_0 e^{j\varphi} e^{j(\omega t - k_0(z-e) - n_y k_0 e)} \end{cases}$
 $\vec{E}_{trans} = E_0 (\hat{u}_x + \alpha e^{j(n_x - n_y)k_0 e + \varphi} \hat{u}_y) e^{j(\omega t - k_0(z-e))}$

4) $\Delta \varphi = (n_x - n_y) k_0 \cdot e = (n_x - n_y) \frac{2\pi}{\lambda} e$
ou $\Delta \varphi = \frac{2\pi}{\lambda} \delta \Rightarrow \delta = e(n_x - n_y)$ (modulo 2π)

5) Lame - demi-onde : $\varphi = \pi + p 2\pi$ (modulo 2π)
Lame quand - onde $\varphi = \frac{\pi}{2} + p 2\pi$.

$\Rightarrow$ demi-onde : $\delta_{\frac{\lambda}{2}} = \frac{\lambda}{2} (1 + 2p)$

$\Rightarrow$ quart d'onde : $\delta_{\frac{\lambda}{4}} = \frac{\lambda}{4} (1 + 4p)$

①

c) $\lambda = 1, p = 0 \rightarrow$ polarisation circulaire rectiligne
cette onde a la sortie d'une lame $\frac{\lambda}{4} \rightarrow$ devient circulaire

- 110 " " " , $\frac{\lambda}{2} \rightarrow$ câble rectiligne
par une lame demi on de onde
 $\lambda = 1,80$ et $\rho = 100$

$$S: \frac{\lambda}{2} (1 + \varepsilon p) \quad \text{et } p = 100$$

$$0,62 \cdot 10^{-6} < \lambda < 0,609 \cdot 10^{-6}$$

$$\frac{0,6 \times 10^{-6} \times 201}{2} < 8 < \frac{0,603 \times 10^{-6} (201)}{2}$$

$$0,67 \cdot 10^{-2} < e < 0,68 \cdot 10^{-2}$$

$$0,67 \text{ nm} < e < 0,68 \text{ nm}$$

EX 102

$$1) \quad y(0, t) = 0, \quad y(l, t) = 0$$

2) $\sqrt{\frac{T_2}{\rho}}$ en $m \cdot s^{-1}$

$$\left[\sqrt{\frac{T_0}{\rho}} \right] = \left[\frac{2g \cdot m \cdot s^{-2}}{2g \cdot m^{-1}} \right]^{\frac{1}{2}} = m \cdot s^{-1} \text{ vibrate}$$

3).

2

RFP $\rho \frac{\partial^2 y}{\partial t^2} = T (x+dx, t) - T(x, t)$

projecting $\left\{ \begin{aligned} \rho \frac{\partial^2 y}{\partial t^2} &= T_y (x+dx, t) - T_y (x, t) \\ 0 &= T_x (x+dx) - T_x (x, t) \end{aligned} \right.$

$\Rightarrow T_x (x+dx) = T_x (x) = T_0 = \text{cte.}$

$\tan \alpha \approx \alpha = \frac{dy}{dx} = \frac{T_y}{T_x}$

$\Rightarrow \rho \frac{\partial^2 y}{\partial t^2} = T_0 \frac{\partial^2 y}{\partial x^2}$

(A.N) $\frac{T_0}{\rho} = c^2 = 200 \text{ m s}^{-2}$

4) $y(x, t) = y_0 \sin \left| \frac{n\pi x}{l} \right| \sin(n\omega_0 t)$

$\frac{\partial^2 y}{\partial x^2} = -\frac{n^2 \pi^2}{l^2} y_0 \sin \left(\frac{n\pi x}{l} \right) \sin(n\omega_0 t)$

$\frac{\partial^2 y}{\partial t^2} = -n^2 \omega_0^2 y_0 \sin \left(\frac{n\pi x}{l} \right) \sin(n\omega_0 t)$

$\frac{\partial^2 y}{\partial t^2} - \frac{T_0}{\rho} \frac{\partial^2 y}{\partial x^2} = -n^2 \omega_0^2 + c^2 \frac{n^2 \pi^2}{l^2} = 0$

$\left[\omega_0 = \frac{\pi c}{l} \right]$

$x=0, \rightarrow y=0$

$x=l \rightarrow y=0$

(3)

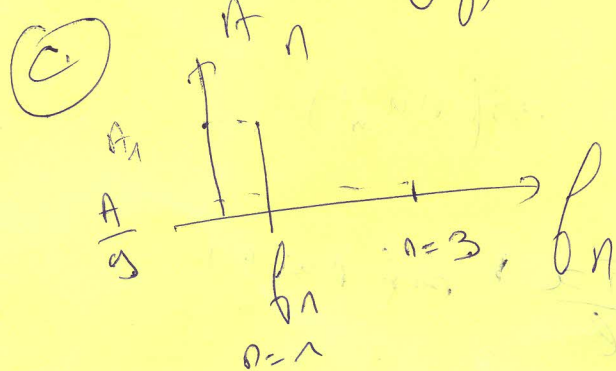
$$b) \quad t = \frac{2\pi}{\omega_n} \quad \left| \begin{array}{l} y(x, t) = y_{0n} \sin\left(\frac{n\pi x}{l}\right) \sin\left(n\omega_0 \frac{2\pi}{\omega_n}\right) = 0 \\ y(x, t) = y_{0n} \sin\left(\frac{n\pi x}{l}\right) \sin\left(n\omega_0 \frac{2\pi}{\omega_n}\right) (n=2) \\ y(x, t) = y_{0n} \sin\left(\frac{n\pi x}{l}\right) \sin\left(n\omega_0 \frac{2\pi}{\omega_n}\right) (n=3) \end{array} \right.$$

$$r = \frac{2\pi}{2\omega} \quad \left| \begin{array}{l} y(x, t) = y_{0n} \sin\left(\frac{n\pi x}{l}\right) \sin\left(\frac{\pi n\omega_0}{2\omega}\right) \\ = y_{0n} \sin\left(\frac{n\pi x}{l}\right) = y_{0n} \sin\left(\frac{2\pi x}{2l}\right) \end{array} \right.$$

$$a) \quad y(x, t) = \sum_{n=1}^{\infty} y_n(x, t) = a_n \cos(\omega_0 t) \sin\left(\frac{n\pi x}{l}\right)$$

$$\omega_n = (2p+1)\omega_0 = 2\pi f_n$$

$$b_n = (2p+1) b_0$$



b) f_n = le son le plus intense

c) Microphone doit être filtre passe-bas

d)

EX 103 :

1) RFD:

$$m \frac{d^2 U_n}{dt^2} = -\alpha (U_n - U_{n-1}) - \alpha (U_n - U_{n+1})$$

$$m \frac{d^2 U_n}{dt^2} = \alpha (U_{n+1} + U_{n-1} - 2U_n)$$

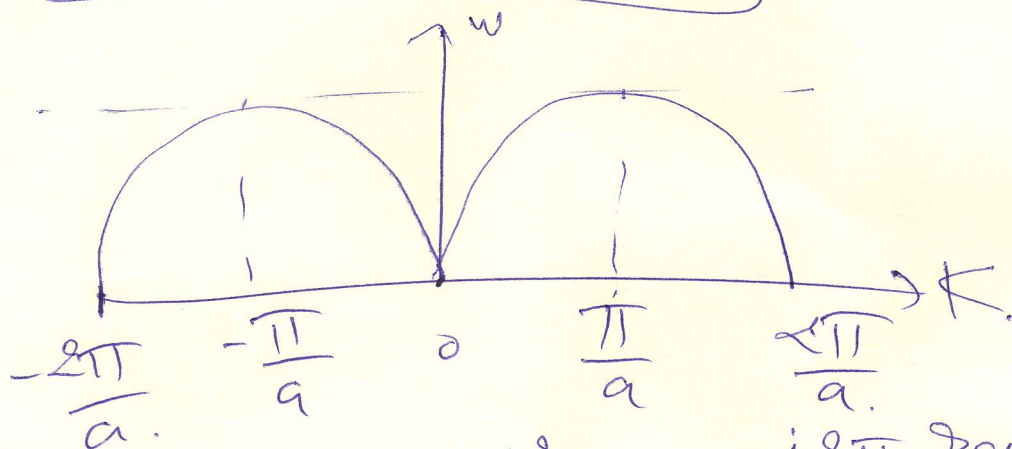
2) U : amplitude de la vibration
 k : nombre d'onde
 ω : pulsation.

3) $U_n = U \exp(j(nka - \omega t))$

$$m(-\omega^2)U_n = \alpha U_n (e^{jka} + e^{-jka} - 2)$$

$$-m\omega^2 = 2\alpha (\cos(ka) - 1)$$

$$\boxed{\omega^2 = \frac{4\alpha}{m} \sin^2\left(\frac{ka}{2}\right)} \text{ eq de dispersion.}$$



4) $\frac{U_{n+1}(t)}{U_n(t)} = e^{jka} = e^{j2\pi \frac{2a}{2\pi a}}$: période de $\frac{2\pi}{a}$.

$$-\frac{\pi}{a} \leq k \leq \frac{\pi}{a}$$



$$\left. \begin{array}{l} \text{pour } z = +\frac{\pi}{a} \\ z = -\frac{\pi}{a} \end{array} \right\} \text{ en opposition de phase}$$

2) Transparence : conditions aux limites
 la relation de dispersion ne donne pas
 de discrétisation des modes propres.

$$\omega = \sqrt{\frac{k\alpha}{m}} \left| \sin\left(\frac{ka}{2}\right) \right|$$

$$v_\varphi = \frac{\omega}{k} = \sqrt{\frac{k\alpha}{m}} \left| \frac{\sin\left(\frac{ka}{2}\right)}{\frac{ka}{2}} \right|$$

$$v_g = \frac{d\omega}{dk} = a \sqrt{\frac{\alpha}{m}} \cos\left(\frac{ka}{2}\right)$$

$$\text{si } z = \pm \frac{\pi}{a}$$

$$v_g = 0$$